New Questions for Junior High Number Sense (2005)

This document contains information on some of the new tricks that will be appearing on the 2005 District and State Number Sense tests. Some of these tricks are not new and students familiar with the Texas Math and Science Coaches Association (TMSCA) contests will recognize many of the following topics.

The new tricks are sectioned in the order that they will appear on the test. Some of the new tricks are given implicitly. For the others, the student is encouraged to search for an easy mental math formula or procedure for working the problem.

MULTIPLICATION BY 51. [#20-40] To multiply any even two-digit number by 51, write the 2-digit number on the right side of the answer. Then, divide the number by 2, and write this quotient on the left side of the answer. For example, to multiply 18×51, write 18 on the right side of the answer. Then, $18 \div 2 = 9$. Write 9 on the left side of the answer to produce the product 918.

MULTIPLICATION BY $16\frac{2}{3}$. [#40-60] I will leave you to figure out the trick to this one. I will, however, let you know that the numbers that the test will ask you to multiply by $16\frac{2}{3}$ will always be multiples of 6.

FRACTIONS IN THE FORM $\frac{1}{n-1}-\frac{1}{n}+\frac{1}{n+1}$. [#40-60] To see where the trick for this problem comes from, use algebra to manipulate the fractions by getting a common denominator and adding. So,

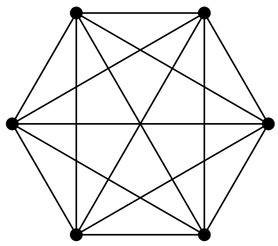
$$\begin{aligned}
\frac{1}{n-1} - \frac{1}{n} + \frac{1}{n+1} &= \frac{1}{n-1}\cdot\frac{n(n+1)}{n(n+1)} - \frac{1}{n}\cdot\frac{(n-1)(n+1)}{(n-1)(n+1)} + \frac{1}{n+1}\cdot\frac{n(n-1)}{n(n-1)} \\
&= \frac{n^2+n-(n^2-1)+n^2-n}{n(n-1)(n+1)} \\
&= \frac{n^2+1}{n(n-1)(n+1)}
\end{aligned}$$

For example, if the problem asks for $\frac{1}{2}-\frac{1}{3}+\frac{1}{4}$, then $n = 3$ (the middle denominator), and by the formula we derived above, the sum is $\frac{3^2+1}{(2)(3)(4)} = \frac{10}{24} = \frac{5}{12}$.

DISTINCT LINES DRAWN USING THE VERTICES OF A REGULAR POLYGON [#60-80] This problem asks you to find the number of distinct (different) lines that can be drawn using the vertices of a regular polygon with n sides as the points on the line. Since the polygon is *regular*, the line between any two vertices will not be part of any other line drawn between other vertices. In other words, using a regular polygon ensures that no three vertices are *collinear*.

Next, to compute the number, you have n vertices from which you must choose 2 points for each line. Therefore, there are $C(n, 2) = n(n-1)/2$ distinct lines.

For example, to find the number of distinct lines that can be drawn using the vertices of a regular hexagon, use $n = 6$, since a hexagon has 6 sides (and thus, 6 vertices). Then, $C(6, 2) = 6 \cdot (6-1)/2 = 15$.



Practice Questions – The following practice questions cover the above examples and should be used to guide your inquiries into the new types of questions to be asked on the number sense tests.

1. $24 \times 51 =$
2. $42 \times 51 =$
3. $32 \times 51 =$
4. $14 \times 51 =$
5. $\frac{1}{4} - \frac{1}{5} + \frac{1}{6} =$
6. $\frac{1}{9} - \frac{1}{10} + \frac{1}{11} =$
7. $\frac{1}{5} - \frac{1}{6} + \frac{1}{7} =$
8. $\frac{1}{12} - \frac{1}{11} + \frac{1}{10} =$
9. $18 \times 16\frac{2}{3} =$
10. $72 \times 16\frac{2}{3} =$
11. $24 \times 16\frac{2}{3} =$
12. $612 \times 16\frac{2}{3} =$
13. How many distinct lines can be drawn using the vertices of a square?
14. How many distinct lines can be drawn using the vertices of a regular pentagon?
15. How many distinct lines can be drawn using the vertices of a regular decagon?
16. How many distinct lines can be drawn using the vertices of a regular 40-gon?

This document was prepared by Doug Ray for PSIA, 2004-2005. If you have any questions about the material presented, please email `doug@academicmeet.com`.