

## New Questions for Junior High Number Sense (2007)

This document contains information on some of the new tricks that will be appearing on the 2007 District and State Number Sense tests. Some of these tricks are not new and students familiar with the Texas Math and Science Coaches Association (TMSCA) contests will recognize many of the following topics.

The new tricks are sectioned in the order that they will appear on the test. Some of the new tricks are given implicitly. For the others, the student is encouraged to search for an easy mental math formula, procedure for working the problem, or information on the topic.

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**MULTIPLICATION BY  $6\frac{1}{4}$  [#20-40]** Since  $6\frac{1}{4} = \frac{25}{4} = \frac{100}{16}$ , the trick to multiplying by  $6\frac{1}{4}$  is to divide the number by 16 first, then multiply by 100.

For example, to multiply  $48 \times 6\frac{1}{4}$ , divide  $48 \div 16 = 3$ , and then multiply  $3 \times 100 = 300$ .

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**SUM OF  $\frac{r^{n-1} - 1}{r^{n-1}} + \frac{r^n - 1}{r^n} + \frac{r^{n+1} - 1}{r^{n+1}}$  [#40-60]** First of all, to recognize this special sum, you should notice the increasing powers of some number in the denominator. These powers will be in order. Next, you should notice that the numerator is just one less than the denominator in each fraction.

To find the sum, consider the following: each fraction can be split up and the sum re-associated.

$$\begin{aligned}\frac{r^{n-1} - 1}{r^{n-1}} + \frac{r^n - 1}{r^n} + \frac{r^{n+1} - 1}{r^{n+1}} &= \left(1 - \frac{1}{r^{n-1}}\right) + \left(1 - \frac{1}{r^n}\right) + \left(1 - \frac{1}{r^{n+1}}\right) \\ &= 3 - \left(\frac{1}{r^{n-1}} + \frac{1}{r^n} + \frac{1}{r^{n+1}}\right) \\ &= 3 - \left(\frac{r^2 + r + 1}{r^{n+1}}\right)\end{aligned}$$

Notice that the fraction  $\frac{r^2 + r + 1}{r^{n+1}}$  does not reduce (why?) and that  $r^2 + r + 1 < r^3$  for  $r \geq 2$  (is there a better lower bound?). Since you are subtracting the fraction from 3, it will be easy to write the answer as a mixed number.

For example,  $\frac{7}{8} + \frac{15}{16} + \frac{31}{32} = 3 - \left(\frac{2^2 + 2 + 1}{32}\right) = 3 - \frac{7}{32} = 2\frac{25}{32}$ .

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**PLATONIC SOLIDS [#60-80]** I will leave this information for you to dig up. The five Platonic Solids are tetrahedron, cube (hexahedron), octahedron, dodecahedron, and icosahedron. You should know the number of vertices, edges, and faces of each.

QUADRATICS [#60-80] The emphasis here is the factorization of a quadratic, the number of real roots (or zeros if written as a function), and the sum or product of the roots.

Much of the information about the roots of a quadratic is known even without knowing the roots themselves and is given by the *discriminant*. The discriminant of the quadratic  $ax^2 + bx + c = 0$  is  $D = b^2 - 4ac$ . Notice that this is just part of the Quadratic Formula. If  $D > 0$ , then the quadratic has 2 real distinct roots. If  $D = 0$ , then the quadratic has 1 real double root. If  $D < 0$ , then the quadratic does not have any real roots.

Finally, for the quadratic  $ax^2 + bx + c = 0$ , the sum of the roots is  $-b/a$  and the product of the roots is  $c/a$ .

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PRACTICE QUESTIONS – The following practice questions cover the above examples and should be used to guide your inquiries into the new types of questions to be asked on the number sense tests.

- $6\frac{1}{4} \times 16 =$
- $96 \times 6\frac{1}{4} =$
- $32 \times 6\frac{1}{4} =$
- $4.8 \times 6.25 =$
- $176 \times 6.25 =$
- $\frac{1}{2} + \frac{3}{4} + \frac{7}{8} =$
- $\frac{2}{3} + \frac{8}{9} + \frac{26}{27} =$
- $\frac{4}{5} + \frac{24}{25} + \frac{124}{125} =$
- $\frac{80}{81} + \frac{26}{27} + \frac{8}{9} =$
- $\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} =$
- How many edges does a cube have?
- How many faces does a dodecahedron have?
- How many vertices does a tetrahedron have?
- A Platonic solid has 8 vertices. How many edges does it have?
- A Platonic solid has twice as many edges as vertices. How many faces does it have?
- How many distinct real roots does the quadratic  $x^2 + 8x + 15 = 0$  have?
- Find the discriminant of  $9x^2 + x - 3 = 0$ .
- Find the sum of the roots of  $8x^2 + 24x - 35 = 0$ .
- The product of the roots of  $3x^2 + 17x + 18 = 0$  is
- If  $x^2 - 9x + 20$  factors as  $(x - A)(x - B)$  with  $A < B$ , find  $A$ .

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This document was prepared by Doug Ray for PSIA, 2006-2007. If you have any questions about the material presented, please email [doug@academicmeet.com](mailto:doug@academicmeet.com).