New Questions for Junior High Number Sense (2009)

This document contains information on some of the new tricks that will be appearing on the 2009 District and State Number Sense tests. Some of these tricks are not new and students familiar with the Texas Math and Science Coaches Association (TMSCA) contests will recognize many of the following topics.

The new tricks are sectioned in the order that they will appear on the test. Some of the new tricks are given implicitly. For the others, the student is encouraged to search for an easy mental math formula, procedure for working the problem, or information on the topic.

MULTIPLICATION BY 99 [#20-40]

The trick to multiplying a two-digit number by 99 is to recognize $99 = 100 - 1$. Thus, write down 1 less than the number for the left digits in the answer, and subtract the two-digit number from 100 to get the right digits.

For example, to compute 23×99, write down $23 - 1 = 22$ on the left and $100 - 23 = 77$ on the right. The answer is 2277.

SUM OF $\frac{1}{n} + \frac{1}{n^2} + \frac{1}{n^3}$ [#40-60]

This trick involves adding fractions where the denominators are the first three powers of a number n. To see where the trick comes from, we use the following:

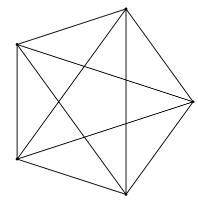
$$\begin{aligned} \frac{1}{n} + \frac{1}{n^2} + \frac{1}{n^3} &= \left(\frac{1}{n} \cdot \frac{n^2}{n^2}\right) + \left(\frac{1}{n^2} \cdot \frac{n}{n}\right) + \left(\frac{1}{n^3}\right) && \text{The common denominator is } n^3 \\ &= \frac{n^2 + n + 1}{n^3} && \text{Combine the fractions} \end{aligned}$$

This equation tells us that the numerator of the sum is found by adding the first two powers of n (listed in the denominators of the problem) and also adding 1. The denominator is simply n^3.

For example, to find the sum $\frac{1}{5} + \frac{1}{25} + \frac{1}{125}$, recognize that $5^2 = 25$ and $5^3 = 125$. This matches our pattern. To find the sum, take $25 + 5 + 1 = 31$ for the numerator and 125 for the denominator. The answer is $\frac{31}{125}$.

Number of distinct diagonals in a regular polygon

[#60-80] A regular polygon with n sides has $\frac{n(n-3)}{2}$ distinct diagonals. Here, "distinct" means that we count each diagonal only once. Thus, the diagonal AF is the same as diagonal FA and is counted once.



To find the number of distinct diagonals of a regular pentagon, use the formula with $n = 5$: $D = \frac{5(5-3)}{2} = 5$.

Sum of first n cube numbers: $1^3 + 2^3 + 3^3 + \ldots + n^3$ [#60-80]

The sum of $1^3 + 2^3 + 3^3 + \ldots + n^3$ is $\left[\frac{n(n+1)}{2}\right]^2$. You may recall that $\frac{n(n+1)}{2}$ is the nth triangular number. Thus, the sum of the first n cube numbers is the square of the nth triangular number.

For example, to find $1^3 + 2^3 + 3^3 + 4^3 + 5^3$, find the 5th triangular number: $\frac{5(6)}{2} = 15$. Square this number: $15^2 = 225$. The answer is 225.

Practice Questions – The following practice questions cover the above examples and should be used to guide your inquiries into the new types of questions to be asked on the number sense tests.

1. $99 \times 45 =$ __________.
2. $81 \times 99 =$ __________.
3. $63 \times 99 =$ __________.
4. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} =$ __________.
5. $\frac{1}{7} + \frac{1}{49} + \frac{1}{343} =$ __________.
6. $\frac{1}{9} + \frac{1}{81} + \frac{1}{729} =$ __________.
7. How many distinct diagonals does a regular hexagon have? __________.
8. How many distinct diagonals does a square have? __________.
9. A regular n-gon has 90 distinct diagonals. Find n. __________.
10. $1^3 + 2^3 + 3^3 + \ldots + 6^3 =$ __________.
11. $1^3 + 2^3 + 3^3 + \ldots + 9^3 =$ __________.
12. $\sqrt{1^3 + 2^3 + 3^3 + \ldots + 20^3} =$ __________.

This document was prepared by Doug Ray for PSIA, 2008-2010. If you have any questions about the material presented, please email `doug@academicmeet.com`.