New Questions for PSIA Junior High Number Sense (2015 and 2016)

This document contains information on some of the new tricks that will be appearing on the 2015 and 2016 Number Sense tests.

The new tricks are sectioned in the order that they will appear on the test. Some of the new tricks are given implicitly. For the others, the student is encouraged to search for an easy mental math formula, procedure for working the problem, or information on the topic.

Special Mixed Number Multiplication [#20-40]

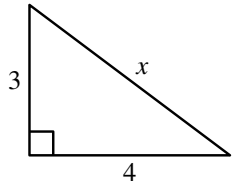
Problems of the type $3\frac{3}{4} \times 4\frac{1}{4}$, $7\frac{5}{6} \times 8\frac{1}{6}$, and $2\frac{3}{5} \times 3\frac{2}{5}$ are all in the same format. Notice that each factor in the problem is equidistant from an integer. For example, $3\frac{3}{4}$ and $4\frac{1}{4}$ are both $\frac{1}{4}$ from 4. Rewriting each factor, we have $3\frac{3}{4} \times 4\frac{1}{4} = (4 - \frac{1}{4}) \times (4 + \frac{1}{4})$. When a product factors in this way, it is called the *difference of two squares*. This factors as $(a - b)(a + b) = a^2 - b^2$. In this sample problem, the product is $4^2 - (\frac{1}{4})^2 = 16 - \frac{1}{16} = 15\frac{15}{16}$. Similarly, $7\frac{5}{6} \times 8\frac{1}{6} = 8^2 - (\frac{1}{6})^2 = 64 - \frac{1}{36} = 63\frac{35}{36}$.

The trick to this problem is to take the middle integer, square it, and subtract one. This will be the integer part of the mixed number answer. For the fraction part, take the fractional-distance from the integer (it will be the fraction with the middle integer from the original problem), square it and subtract it from one.

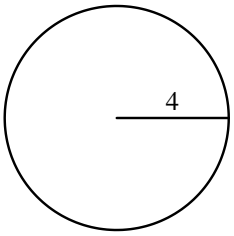
For example, to compute $2\frac{3}{5} \times 3\frac{2}{5}$, take the middle integer, 3, square it to get 9 and subtract one, to get 8. Write 8. Take the fraction that was with the 3, namely $\frac{2}{5}$, square it, to get $\frac{4}{25}$, and subtract it from one, to get $\frac{21}{25}$. The short-cut to subtracting the fraction from one is to subtract the denominator minus the numerator. The answer is $8\frac{21}{25}$.

Geometry Figures [#40-60]

New to the Junior High Number Sense test this year, one of the questions between 40 and 60 will feature a geometric figure and a question about one of the quantities related to this figure. The figure could be any two- or three-dimensional shape and the quantities to solve for could be any side length, interior length, angle, perimeter, area, or volume. Basically any problem that would have been asked for in words will be considered a possibility for this new question type. Although we are considering this type of problem to be a special problem type for these two years, if the results are positive, I will consider this type of problem in future tests without the special topic designation.



Find x. _____



Circumference = _____

Difference of Consecutive Cubes [#60-80]

One of the questions in the last quarter of the test will be to find the difference of consecutive cube numbers. For example, $8^3 - 7^3$ and $17^3 - 16^3$. I will leave it to you to determine a short-cut to computing this difference without actually knowing the cubes and doing the subtraction. As a hint, you should investigate one of algebra's special factorizations known as the *difference of two cubes*.

Inverse Functions [#60-80]

Inverse functions are functions that essentially "undo" a given function. For example, the inverse of the function $f(x) = 3x - 2$ is $f^{-1}(x) = \dfrac{x+2}{3}$ [f^{-1} is read as *f inverse*]. Notice that $f(5) = 3(5) - 2 = 13$ and that $f^{-1}(13) = \dfrac{13+2}{3} = \dfrac{15}{3} = 5$. When you map $x = 5$ with f, you get 13 and when you map 13 with f^{-1}, you get back to 5. The inverse function f^{-1} returned the original x-value that we started with.

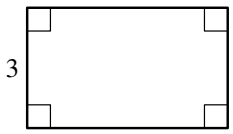
When given a function f that maps x's to y's, the inverse function f^{-1} maps the y's back to the x's. For example, if $f(x) = 5x - 3$ and you are asked to find $f^{-1}(7)$, the 7 is a y-value from the original function. Set $5x - 3$ equal to 7 and solve for x: $5x - 3 = 7$ gives $5x = 10$ and $x = 2$.

Practice Questions – The following practice questions cover the above examples and should be used to guide your inquiries into the new types of questions to be asked on the number sense tests.

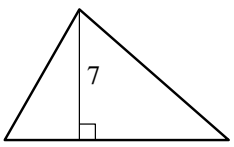
1. $5\frac{3}{4} \times 6\frac{1}{4} =$ _____ (mixed number)

2. $8\frac{6}{7} \times 9\frac{1}{7} =$ _____ (mixed number)

3. $3\frac{5}{6} \times 4\frac{1}{6} =$ _____ (mixed number)

4. 

Area = _____

5. 

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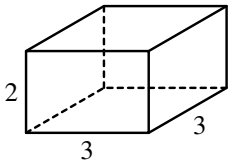
6. 

Volume = _____

7. $5^3 - 4^3 =$ _____

8. $9^3 - 8^3 =$ _____

9. $16^3 - 15^3 =$ _____

10. $21^3 - 20^3 =$ _____

11. If $f(x) = 2x - 1$, then $f^{-1}(11) =$ _____

12. If $f(x) = 4x + 8$, then $f^{-1}(4) =$ _____

13. If $f(x) = \dfrac{x+2}{3}$, then $f^{-1}(1) =$ _____

This document was prepared by Doug Ray for competition, 2015 and 2016. If you have any questions about the material presented, please email doug@academicmeet.com.