# New Questions for PSIA Junior High Number Sense
## (2017 and 2018)

This document contains information on some of the new tricks that will be appearing on the 2017 and 2018 Number Sense tests.

The new tricks are sectioned in the order that they will appear on the test. Some of the new tricks are given implicitly. For the others, the student is encouraged to search for an easy mental math formula, procedure for working the problem, or information on the topic.

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### MULTIPLICATION BY A MULTIPLE OF 111 [#20-40]

Examples of problems of this type are $333 \times 7$ and $555 \times 13$. The trick to easily solve this problem is to shift the number that is multiplied by 111 to the other number. Then, use the trick for multiplying by 111 to solve.

For example, to solve $333 \times 7$, since $333 = 3 \times 111$, shift the 3 to the 7 to make 21. Then, solve $111 \times 21$. The product is 2331.

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### SUM AND DIFFERENCE OF SPECIAL CUBES [#40-60]

Problems of this type will be in the form $(n + 1)^3 - n^3 - n^3 + (n - 1)^3$. For example, $9^3 - 8^3 - 8^3 + 7^3$. The student is encouraged to work out a very easy mental math trick for problems of this type. You might consider working several of these problems out on a calculator and looking for the obvious pattern. Then, try to provide an algebraic proof that your conjecture is correct. Good luck!

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### DECIMAL VALUE OF RECIPROCAL [#60-80]

In these problems, students will be asked to compute the decimal value of a reciprocal to four digits past the decimal point. For example, "The decimal value of $\frac{1}{24}$ is 0. _____ (4 decimal places)." Students will be allowed to truncate or round the last place. There are several techniques students may use to solve these, but they are dependent on the problem at hand.

1. Locate the number of the number line. These problems will be restricted to the reciprocals of numbers from 1 to 100. The reciprocal of any number larger than 10 has a 0 as the first decimal digit. Use that fact to your advantage. There are other numbers that serve as partitions for what the second digit can be. What do you know about the reciprocal of a number between 51 and 100, inclusive?

2. Look for common fractions and use their decimal versions that you may already know.

For example, when solving $\frac{1}{24}$, the first decimal digit is 0 by the first suggest above. Then, you divide $100 \div 24$. The quotient is 4, so write down the 4. Next, you take the remainder from $100 \div 24$, which is also 4 and you would divide $4 \div 24$. This is the same as $1 \div 6$. Students should know the decimal version of $\frac{1}{6}$ is $0.16666\ldots$. This gives us the rest of the decimal digits. The complete answer is 0.0416 or 0.0417.

EXTREME VALUES OF QUADRATICS [#60-80]

A quadratic function is one in the form $f(x) = ax^2 + bx + c$, where $a \neq 0$. The degree (highest power) is 2. The graph of a quadratic function is a parabola. The parabola either opens up (if $a > 0$) or opens down (if $a < 0$). The ones that open up achieve a minimum value at the vertex. The ones that open down achieve a maximum value at the vertex.

For the quadratic function $f(x) = ax^2 + bx + c$, the vertex occurs at the point $\left(-\dfrac{b}{2a}, \; -\dfrac{b^2}{4a} + c\right)$.

EXAMPLE    What is the $x$-coordinate of the vertex of $y = 2x^2 - 3x + 4$?

The $x$-coordinate of the vertex is given by the formula $x = -\dfrac{b}{2a}$. Thus, the answer is $-\dfrac{-3}{2(2)} = \dfrac{3}{4}$.

EXAMPLE    What is the maximum value of $y = -x^2 + 8x - 10$?

The extreme value is the $y$-coordinate of the vertex and is given by the formula $-\dfrac{b^2}{4a} + c$. Thus, the maximum value is $-\dfrac{8^2}{4(-1)} - 10 = 16 - 10 = 6$. Note that this type of question will either ask for the minimum or maximum depending of if it opens up or opens down, respectively.

PRACTICE QUESTIONS – The following practice questions cover the above examples and should be used to guide your inquiries into the new types of questions to be asked on the number sense tests.

1. $444 \times 9 = $ _____

2. $777 \times 8 = $ _____

3. $333 \times 17 = $ _____

4. $888 \times 12 = $ _____

5. $5^3 - 4^3 - 4^3 + 3^3 = $ _____

6. $11^3 - 10^3 - 10^3 + 9^3 = $ _____

7. $16^3 - 15^3 - 15^3 + 14^3 = $ _____

8. $22^3 - 21^3 - 21^3 + 20^3 = $ _____

Write the decimal value of each to 4 decimal places.

9. $\dfrac{1}{16} = 0.$ _____

10. $\dfrac{1}{18} = 0.$ _____

11. $\dfrac{1}{29} = 0.$ _____

12. $\dfrac{1}{85} = 0.$ _____

13. Find the $x$-coordinate of the vertex of $y = x^2 - 12x + 17.$ _____

14. Find the $x$-coordinate of the vertex of $y = -3x^2 - 6x + 10.$ _____

15. What is the minimum value of $y = x^2 - 4x - 12$?
_____

16. What is the maximum value of $y = -2x^2 + 12x + 50$? _____