Junior High Number Sense – Special Topics

Competition in Spring 2019 and Spring 2020

This document contains information on some of the new tricks that will appear on the 2019 and 2020 Number Sense tests.

The new tricks are sectioned in the order that they will appear on the test. Some of the new tricks are given implicitly. For the others, the student is encouraged to search for an easy mental math formula, procedure for working the problem, or information on the topic.

SQUARING NUMBERS CLOSE TO 1000 [#20-40]

A question of this type may ask you to find the value of 983^2, for example. The trick is to consider the difference between 1000 and 983. This result is $1000 - 983 = 17$. For the last 3 digits of the result, square this difference: $17^2 = 289$. For the first 3 digits of the result, subtract twice the difference from 1000: $1000 - 34 = 966$. The answer is 966,289.

SPECIAL MIXED NUMBER MULTIPLICATION [#40-60]

Problems of this type will be in the form similar to $6\frac{1}{7} \times 7\frac{1}{6}$ and $5\frac{1}{9} \times 9\frac{1}{5}$. The student is encouraged to work out an easy mental math trick for problems of this type. You might consider working several of these problems out and looking for the obvious pattern. Then, try to provide an algebraic proof that your conjecture is correct. Good luck!

NUMBER OF LINES PASSING THROUGH n POINTS [#60-80]

This question asks for the number of lines that pass through a set of n points, no 3 of which are collinear. This last part ensures that all lines are different. Since it takes 2 points to create a line, the number of lines through n points is $C(n, 2)$, read "n choose 2". This value is the combination of choosing 2 points from the set of n points without regard to order. The order is unimportant since line AB is the same as line BA and we do not want to count these twice. The formula for $C(n, 2) = \dfrac{n(n-1)}{2}$.

EXAMPLE   How many lines can be drawn through 5 points, no 3 of which are collinear?
SOLUTION   The answer is $C(5, 2) = 5(4)/2 = 10$.

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nTH PENTAGONAL NUMBER [#60-80]

The pentagonal numbers are the numbers in the sequence of the number of dots it takes to form regular pentagons with sides with 1 dot, then 2 dots, then 3 dots, and so on. There are several ways to find P_n, the nth pentagonal number.

1. Explicit Formula: $P_n = \dfrac{n(3n-1)}{2}$

2. $P_n = S_n + T_{n-1}$, where S_n is the nth square number and T_{n-1} is the $(n-1)$th triangular number

P_1 P_2 P_3 P_4
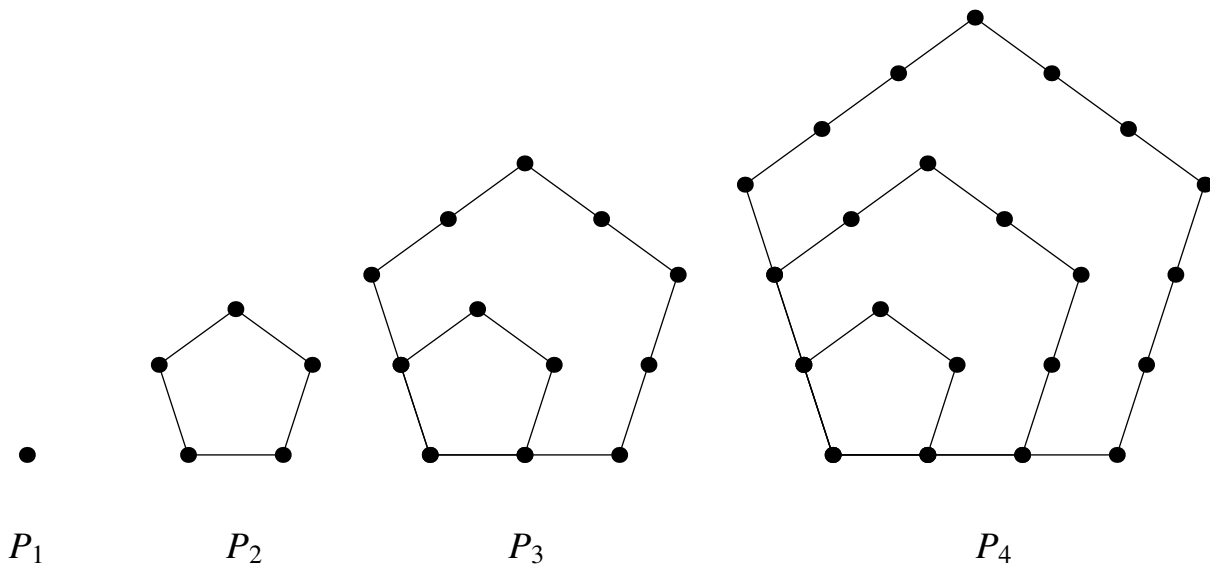
For example, what is the 10th pentagonal number? $P_{10} = S_{10} + T_9 = 10^2 + \dfrac{9(10)}{2} = 100 + 45 = 145$.

There are many other formulas as well. Find one that you like!

PRACTICE QUESTIONS – The following practice questions cover the above examples and should be used to guide your inquiries into the new types of questions to be asked on the number sense tests.

1. $997^2 =$ _____

2. $992^2 =$ _____

3. $989^2 =$ _____

4. $978^2 =$ _____

5. $8\dfrac{1}{3} \times 3\dfrac{1}{8} =$ _____ (mixed number)

6. $6\dfrac{1}{5} \times 5\dfrac{1}{6} =$ _____ (mixed number)

7. $12\dfrac{1}{7} \times 7\dfrac{1}{12} =$ _____ (mixed number)

8. $3\dfrac{1}{6} \times 6\dfrac{1}{3} =$ _____ (mixed number)

9. How many lines can be drawn through 8 points, no 3 of which are collinear? _____

10. How many lines can be drawn through 20 points, no 3 of which are collinear? _____

11. How many lines can be drawn through 6, points, no 3 of which are collinear? _____

12. How many lines can be drawn through 23 points, no 3 of which are collinear? _____

13. What is the 7th pentagonal number? _____

14. What is the 11th pentagonal number? _____

15. The nth pentagonal number is 35. What is n? _____

16. What is the 15th pentagonal number minus the 14th triangular number? _____